LEES

MELBOURNE AND METROPOLITAN TRAMWAYS BOARD

ENGINEERING DEPARTMENT

PLANNING BRANCH

HORIZONTAL CURVES

GENERAL ARRANGEMENT AND SELECTION

MAY 1966.

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Where changes in track direction occur it is necessary to design the curved track to permit the maximum operating speeds. This requires the optimum arrangement of circular curvature and transition to and from the straight or tangent track.

1. Length of Transition.

Here a distinction should be made between what may be regarded as a desirable length and what is generally accepted as a minimum.

1.1 Minimum Length.

(a) Cant gradient should not be greater than 1/300 (British Railway Practice).

(b) Rate of gain (or loss) of cant should not be greater than 2¼" inches per second. (British Railway Practice).

(c) The build up of the centrifugal force term v^2/R should not be greater than 2 ft. per second^3 . (Reference Plane & Geodetic Surveying Part 1 - Clark).

The corresponding minimum lengths of transitions from straight track to circular curves with 6 inches cant and speeds to give a maximum of 0.1g side thrust are as follows:-

Radius ft.	150	200	300	400	500	600
Max. Speed m.p.h.	22 (19½)	25 (22½)	30 (27)	35 (31½)	39 (35½)	42 (39)
Lth. to give cant gradient = 1/300	100 / 150	100 / 150	100 / 150	100 / 150	100 / 150	100 / 150
Lth. to give rate of gain of cant of 2¼" per sec.	-	-	-	-	152	101 / 168
Lth. to give a gain of v^2/R of 2 ft. per sec^3.	-	-	102 -	120 165	140 185	155 200

figures for 4" cant in brackets

1.8

1.2 Desirable length.

(a) Cant gradient should not be less than 1/450 (British Railway Practice).

(b) Rate of gain (or loss) of cant should not be greater than 1½" per second (British Railway Practice).

(c) The build up of the centrifugal force term v^2/R should not be greater than "a" ft. per sec^3 where "a" = 1.75 - 0.0075 V and V is the maximum speed in miles per hour. (Reference Plane and Geodetic Surveying Part 1 - Clark).

This is of course an empirical formula based on a maximum of 1.75 feet per sec^3 and a linear reduction as the maximum speed is raised. It is little more than a compromise whereas British Railway recommendations are based on a number of properly conducted trials.

The corresponding desirable lengths of transitions from straight track to circular curves with 6 inches cant and speeds to give a maximum of 0.1g side thrust are as follows:-

Radius ft.	150	200	300	400	500	600
Max. Speed m.p.h.	19½ 22	22½ 25	27 30	31½ 35	35½ 39	39 42
Lth. to give cant gradient = 1/450	150 225	150 225	150 225	150 225	150 225	150 225
Lth. to give rate of gain of cant of 1½" per sec.	-	-	175	205	230	152 250
Lth. to give a gain of v^2/R of 1.75 - 0.0075 V	-	-	100 133 185	120 160 220	188 255	210 ~~280~~ 270

figures for 4" cant shown altered

1.3 Design length.

In locations where the radius of curvature is severely restricted and therefore maximum speed, the length of transition should be kept close to the minimum. Hence to adopt transition lengths for preliminary design based on the curve for increase of v^2/R not greater than a = 1.75 - 0.0075V and a cant gradient of not less than 1/300 should be satisfactory - refer Diagram No. 1. This gives the following:-

Radius of Circular Curve - ft.	150	200	250	300	350	400	450	500	600
Length of transition ft.	100 150	115 160	170	135 185	200	165 220	235	185 250	215 280 275

2. Overall dimensions of transition.

Horizontal transitions for underground tramways should be designed such that the radius of curvature varies strictly inversely as the length of transition. The Euler Spiral meets this requirement and the first term of its "expansion" gives the cubic spiral

$$y = \frac{l^3}{6RL}$$

If the angle of the transition is not great such that it may be assumed that $l = x$ we get the cubic parabola

$$y = \frac{x^3}{6RL}$$

Further work will however be based on the cubic spiral unless the desired accuracy requires the use of further terms of the above expansion; which of course will be necessary in the ultimate design.

2.1 Shift. Refer Diagram No. 2.

The shift tangent is defined as the tangent to the extended circular arc portion of the curve which is parallel to the straight track tangent.

The shift is defined as the distance the shift tangent is away from the straight track tangent.

2.2 Procedure for arriving at overall dimension.

The procedure is one of trial and correction.

Proceed thus:-

(a) Assume a value for the minimum radius R

(b) select corresponding length L of transition refer 1.3 above

(c) determine the transition angle ϕ which for the Euler Spiral is given by the simple expression

$$\phi = \frac{L}{2R} \quad \text{radians}$$

(d) determine from table 1 the corresponding value of S/L for the Euler Spiral where S = shift and hence find S (Reference Plane and Geodetic Surveying Part 1 - Clark).

(e) locate the centre of the circular arc and hence locate the circular arc

(f) determine from table 1 the corresponding value of C/L for the Euler Spiral where C = extension and hence find C.

Note: C is approximately equal to half L.

The transition curve may then be set out using the expansions.

$$y = \frac{l^3}{3(2RL)} - \frac{l^7}{7.3.2.(2RL)^3} + \frac{l^{11}}{11.5.4.3.2.(2RL)^5} -$$

$$x = l - \frac{l^5}{5.2.(2RL)^2} + \frac{l^9}{9.4.3.2.(2RL)^4} -$$

where l = length of arc corresponding to x, y.

If clearances are such as to permit a larger value for the minimum radius R repeat until a curve arrangement is obtained which will permit the maximum speed considered desirable

References.

"Preliminary Investigation of Curve Design for Underground Tramways" Part II - Planning Branch April 1966.

"Recent Developments in Railway Curve Design" Proceedings of Inst. of Civil Engineers October 1952.

"Plane & Geodetic Surveying" - Part 1 - Clark.

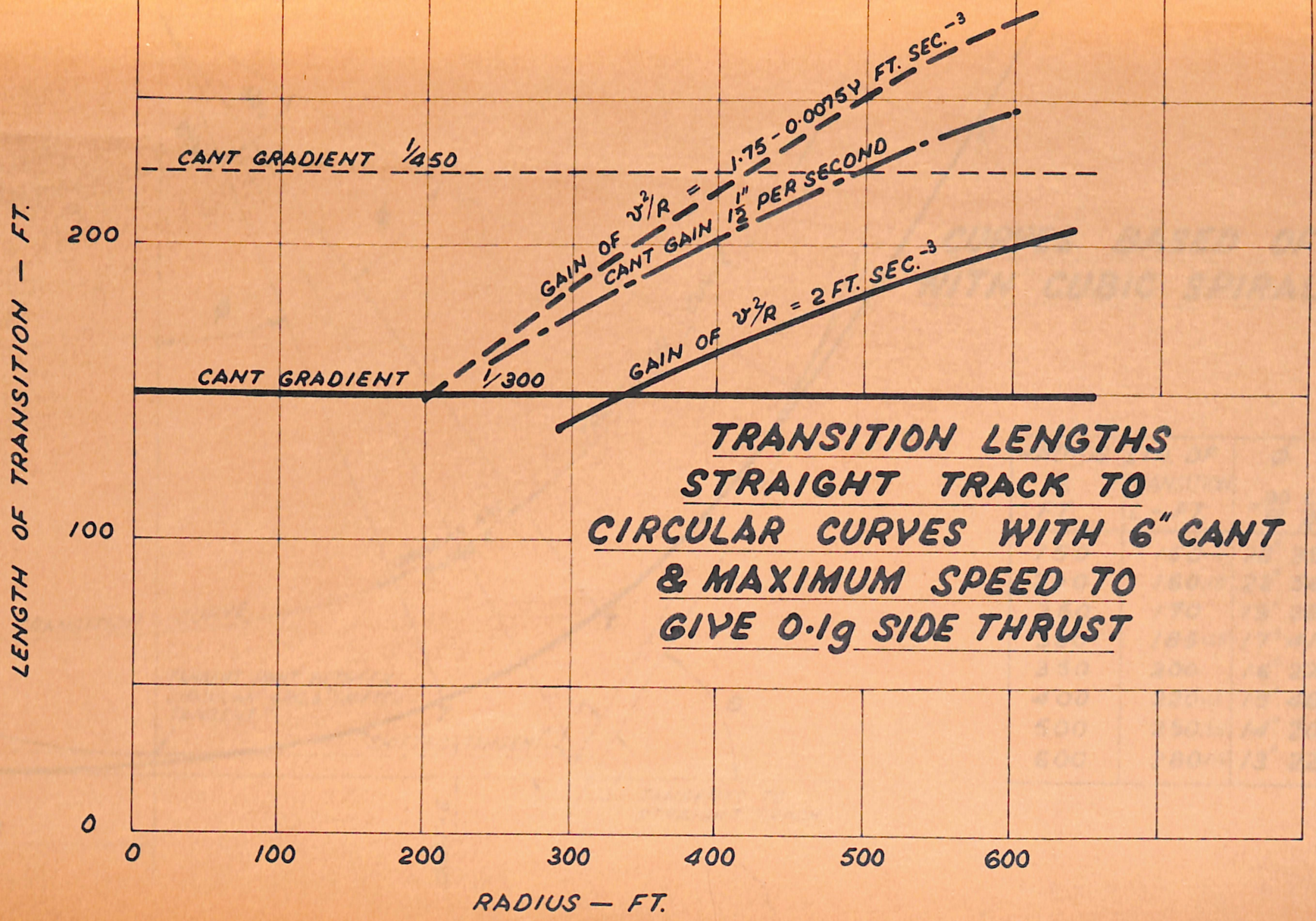
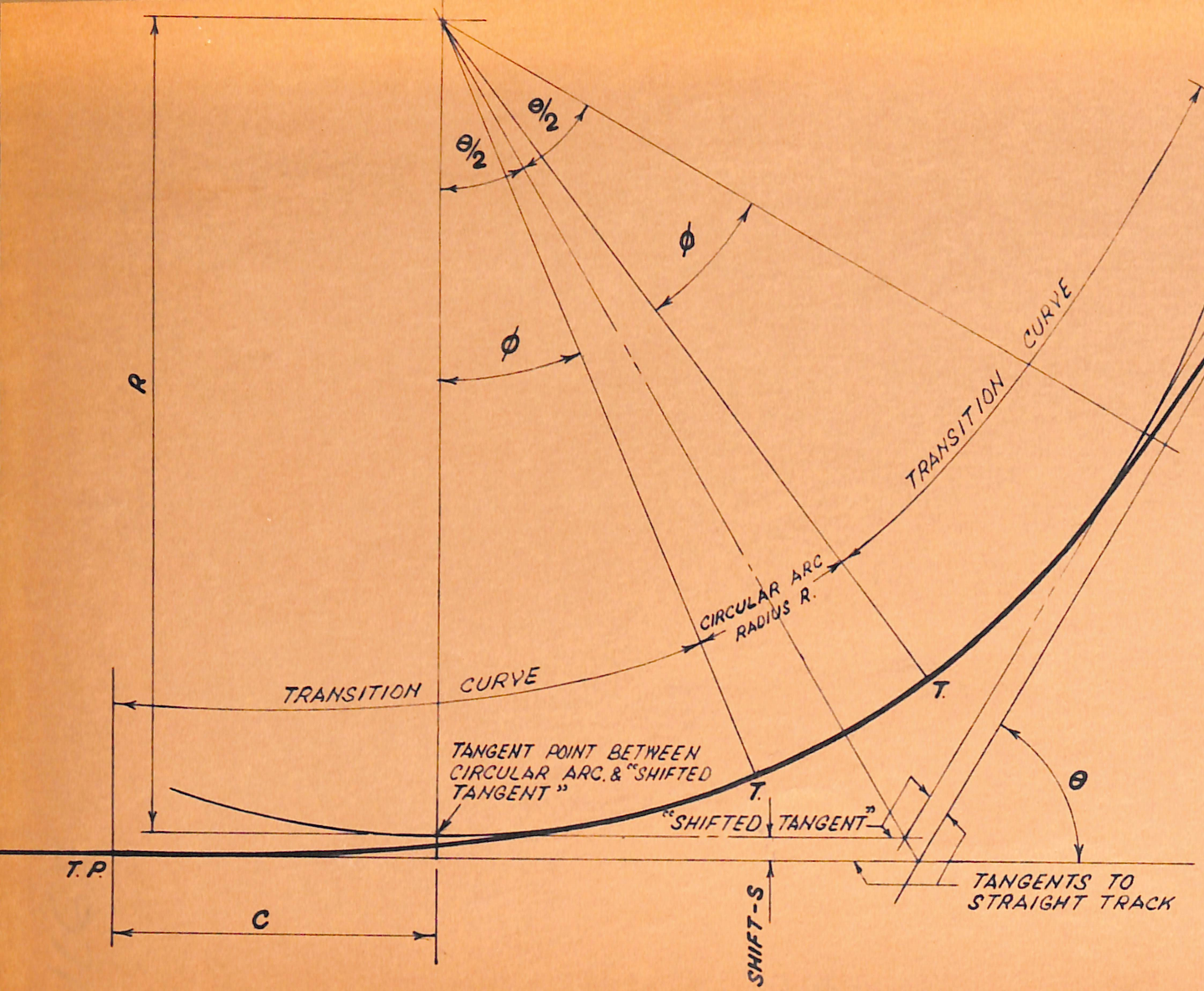


DIAGRAM No 1

5/15/66



CURVES BASED OF CIRCULAR ARCS WITH CUBIC SPIRAL TRANSITIONS

RADIUS R. FT.	LTH. OF TRANSITION L - FT.	$\phi = \frac{90}{\pi}\frac{L}{R}$	SHIFT S. FT.	EXTENSION C. FT.
150	150/100	28° 39′	6·20	74·38
200	160/115	22° 56′	5·31	79·58
250	170	19° 29′	4·80	84·67
300	185/135	17° 41′	4·74	92·21
350	200	16° 22′	4·75	99·73
400	220/165	15° 46′	5·03	109·72
500	250/185	14° 20′	5·20	124·74
600	280/215	13° 22′	5·43	139·75

DIAGRAM N°2